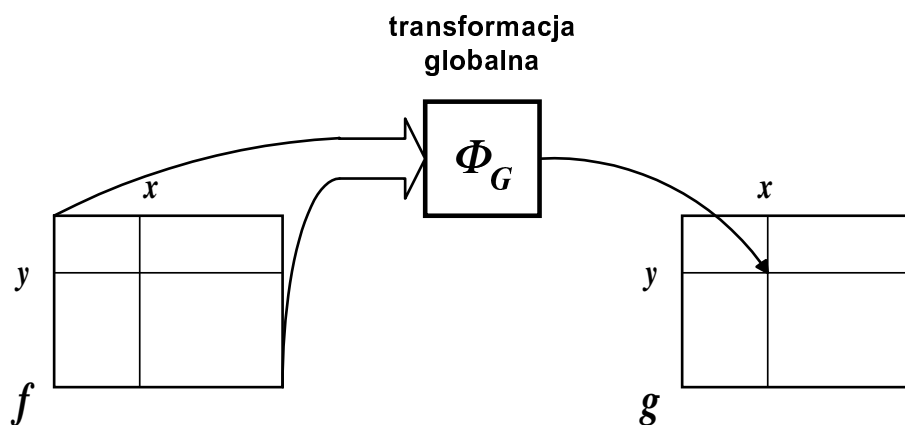
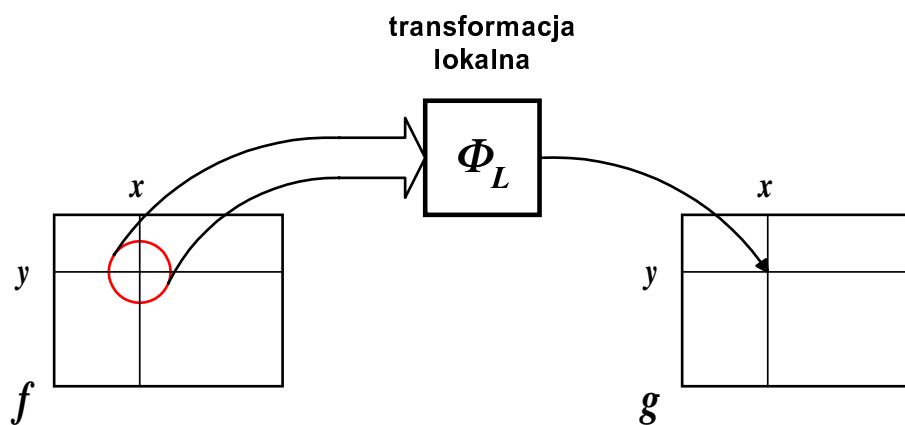
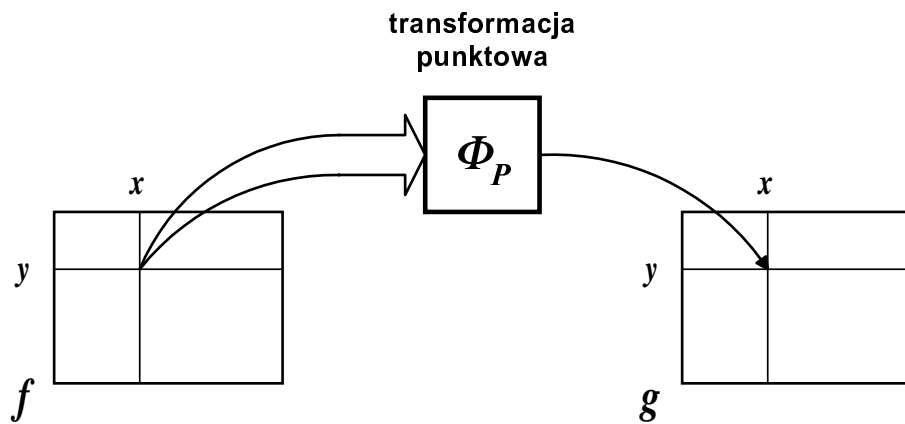


Typy transformacji obrazów



Liniove transformacje globalne obrazów

$$\mathbf{f}_{H \times W} = \begin{bmatrix} f(0,0) & f(0,1) & \dots & f(0,W-1) \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ f(H-1,0) & f(H-1,1) & \dots & f(H-1,W-1) \end{bmatrix}$$

$$\mathbf{F}_{H \times W} = \mathbf{P}_{H \times H} \mathbf{f}_{H \times W} \mathbf{Q}_{W \times W}; \det \mathbf{P} \neq 0, \det \mathbf{Q} \neq 0$$

$$F(u, v) = \sum_{m=0}^{H-1} \sum_{n=0}^{W-1} P(u, m) f(m, n) Q(n, v)$$

$$\mathbf{f}_{H \times W} = \mathbf{P}_{H \times H}^{-1} \mathbf{F}_{H \times W} \mathbf{Q}_{W \times W}^{-1}$$

Ogólne własności liniowych transformacji globalnych

Separowalność:

$$\mathbf{F} = (\mathbf{P} \mathbf{f}) \mathbf{Q} = \mathbf{P} (\mathbf{f} \mathbf{Q})$$

Dla \mathbf{P} i \mathbf{Q} rzeczywistych, ortogonalnych i symetrycznych oraz dla \mathbf{P} i \mathbf{Q} zespolonych, hermitowskich i unitarnych:

$$\mathbf{F} = \mathbf{P} \mathbf{f} \mathbf{Q}$$

$$\mathbf{f} = \mathbf{P} \mathbf{F} \mathbf{Q}$$

Przypomnienie:

$$\mathbf{A} \text{ jest symetryczna} \Leftrightarrow \mathbf{A}^T = \mathbf{A}$$

$$\mathbf{A} \text{ jest ortogonalna} \Leftrightarrow \mathbf{A}^T \mathbf{A} = \mathbf{1}$$

$$\mathbf{A} \text{ jest hermitowska} \Leftrightarrow \mathbf{A}^T = \mathbf{A}^*$$

$$\mathbf{A} \text{ jest unitarna} \Leftrightarrow \mathbf{A}^T \mathbf{A}^* = \mathbf{1}$$

Dyskretna transformacja Fourier-a (DFT)

$$\mathbf{E}_K[m, n] = \frac{1}{\sqrt{K}} e^{-j\frac{2\pi}{K}mn}; m, n \in Z_0^{K-1}$$

$$\mathbf{E}_K^{-1}[m, n] = \frac{1}{\sqrt{K}} e^{j\frac{2\pi}{K}mn}; m, n \in Z_0^{K-1}$$

$$\mathbf{F}_{H \times W} = \mathbf{E}_H \mathbf{f}_{H \times W} \mathbf{E}_W$$

$$F(u, v) = \frac{1}{\sqrt{HW}} \sum_{m=0}^{H-1} \sum_{n=0}^{W-1} f(m, n) e^{-j2\pi(\frac{mu}{H} + \frac{nv}{W})}$$

$$\mathbf{f}_{H \times W} = \mathbf{E}_H^{-1} \mathbf{F}_{H \times W} \mathbf{E}_W^{-1}$$

$$f(m, n) = \frac{1}{\sqrt{HW}} \sum_{u=0}^{H-1} \sum_{v=0}^{W-1} F(u, v) e^{j2\pi(\frac{mu}{H} + \frac{nv}{W})}$$

Własności transformacji Fourier-a

$$F(u, -v) = F(u, W - v)$$

$$F(-u, v) = F(H - u, v)$$

$$F(-u, -v) = F(H - u, W - v)$$

$$F(aH + u, bW + v) = F(u, v); a, b \in \mathbb{Z}$$

$$f(-m, n) = f(H - m, n)$$

$$f(m, -n) = f(m, W - n)$$

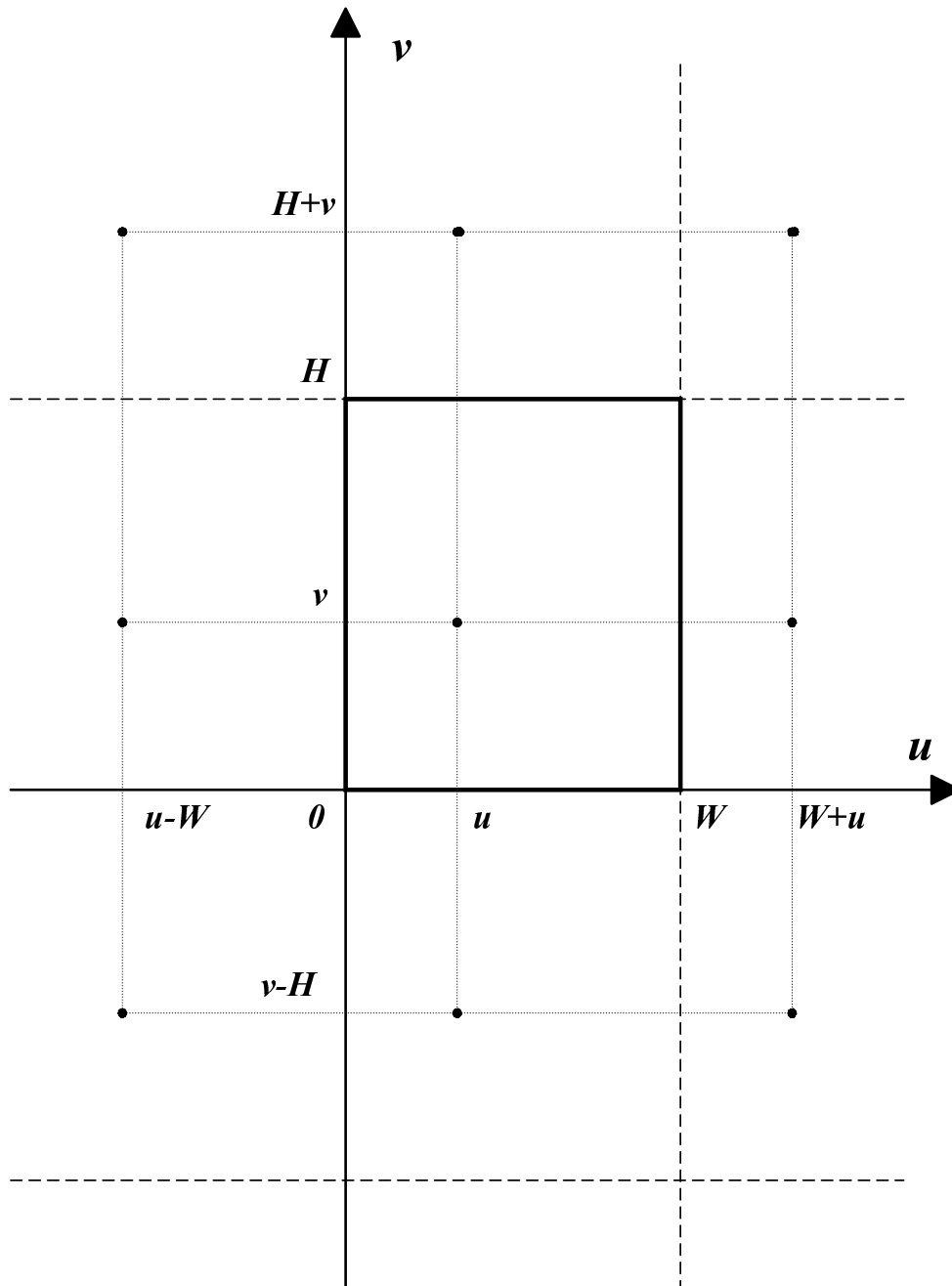
$$f(-m, -n) = f(H - m, W - n)$$

$$f(aH + m, bW + n) = f(m, n); a, b \in \mathbb{Z}$$

$$\mathbf{g} = \mathbf{f} * \mathbf{h} \Leftrightarrow \mathbf{G} = \mathbf{G} \mathbf{H}$$

$$\mathbf{G} = \mathbf{F} * \mathbf{H} \Leftrightarrow \mathbf{g} = \mathbf{f} \mathbf{h}$$

Cykliczność transformacji Fourier-a



Szybka transformacja Fourier-a (FFT)

$$K = 2^b, \quad z = e^{-j\frac{2\pi}{K}}$$

$$\sqrt{K}F(u) = G_b = \sum_{k=0}^{K-1} f(k)z^{ku}$$

$$G(u) = \sum_{m=0}^{\frac{K}{2}-1} (f(2m)z^{2mu} + f(2m+1)z^{(2m+1)u})$$

$$M = \frac{N}{2}, \quad q = z^2, \quad f(2m) = f_e(m), \quad f(2m+1) = f_o(m)$$

$$G(u) = \sum_{m=0}^{M-1} f_e(m)q^{mu} + z^u \sum_{m=0}^{M-1} f_o(m)q^{mu}$$

...

aż do $M=1$

Algorytm motylkowy FFT

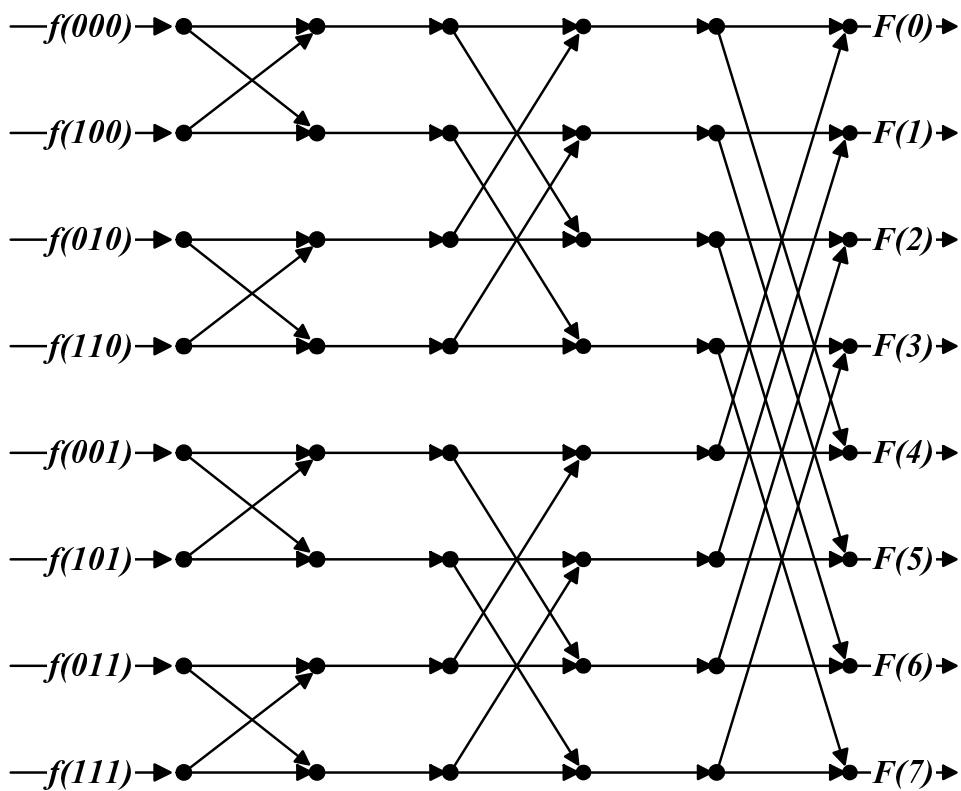
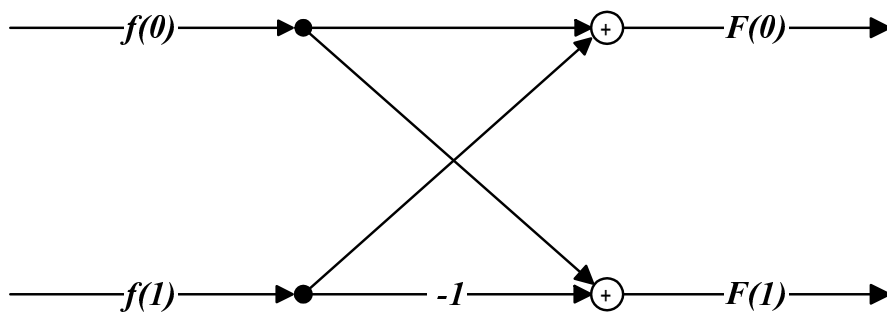
$$N = 2, M = 1$$

$$F(0) = \sum_{m=0}^0 f_e(0)q^0 + z^0 \sum_{m=0}^0 f_o(0)q^0 = f(0) + f(1)$$

$$F(1) = \sum_{m=0}^0 f_e(0)q^0 + z^1 \sum_{m=0}^0 f_o(0)q^0 = f(0) - f(1)$$

$$C(K) = cK \lg_2 K + O(K)$$

Idea algorytmu motylkowego

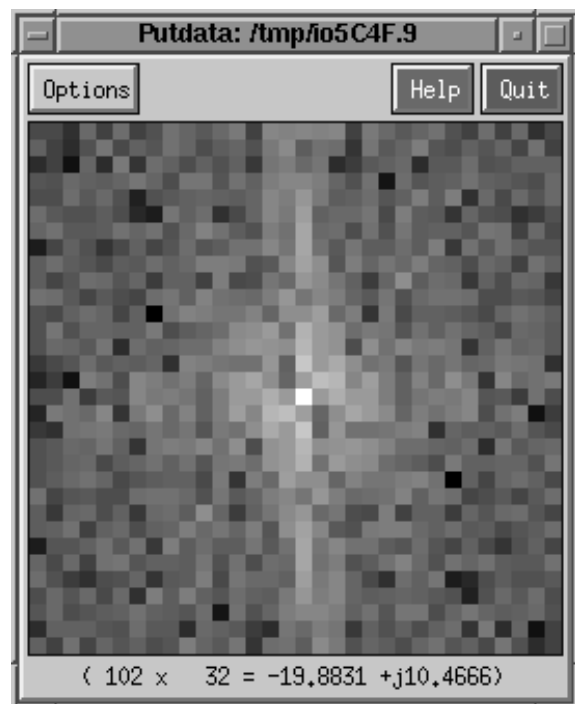


Przykładowy obraz i jego transformata

obraz



transformata



Dyskretna transformacja kosinusowa (DCT)

$$\mathbf{C}_K[m, n] = \frac{\kappa_m}{\sqrt{K}} \cos\left(\frac{m\pi}{K}(n + 0.5)\right)$$

$$\kappa_m = \begin{cases} 1 & \Leftrightarrow m = 0 \\ \sqrt{2} & \Leftrightarrow m \in \mathbb{Z}_1^{K-1} \end{cases}$$

$$\mathbf{C}_K \mathbf{C}_K^T = \mathbf{1}_K$$

$$K = 2L; \mathbf{x} \in \mathbf{R}^K; \mathbf{y} \in \mathbf{R}^K$$

$$\mathbf{X} = \mathbf{C}_K \mathbf{x}; \mathbf{Y} = \mathbf{E}_K \mathbf{y}$$

$$\mathbf{y}_l = \begin{cases} x_{2l} & \Leftrightarrow l = 0, 1, \dots, \frac{K}{2} - 1 \\ x_{2K-2l-1} & \Leftrightarrow l = \frac{K}{2}, \dots, K - 1 \end{cases}$$

$$X_l = \begin{cases} \kappa_l \operatorname{Re}(e^{\frac{-j\pi}{2K}l} Y_l) & \Leftrightarrow l = 0, 1, \dots, \frac{K}{2} \\ -\kappa_l \operatorname{Im}(e^{\frac{-j\pi}{2K}l} Y_{K-l}) & \Leftrightarrow l = \frac{K}{2} + 1, \dots, K - 1 \end{cases}$$

Macierz Hadamard-a

- (h1) $\mathbf{H}_K = \mathbf{H}_K^T$
- (h2) $\mathbf{H}_K[m, n] \in \{-1, 1\}$; $m, n = 0, 1, \dots, K-1$
- (h3) $\mathbf{H}_K \mathbf{H}_K^T = c \mathbf{1}_K$

$$\mathbf{H}_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Własności macierzy Hadamard-a:

$$\mathbf{H}_K \mathbf{H}_K^T = K \mathbf{1}_K$$

$$\mathbf{H}_{2K} = \begin{bmatrix} \mathbf{H}_K & \mathbf{H}_K \\ \mathbf{H}_K & -\mathbf{H}_K \end{bmatrix}$$

$$\mathbf{H}_{2K} = \mathbf{H}_2 \otimes \mathbf{H}_K$$

$$\mathbf{H}_K = \underbrace{\mathbf{H}_2 \otimes \mathbf{H}_2 \otimes \dots \otimes \mathbf{H}_2}_{k \text{ razy}} ; K = 2^k$$

$$\mathbf{H}_K[m, n] = (-1)^{\sum_{p=0}^{k-1} m_p n_p} ; m_p : p\text{-ty bit } m$$

Transformacja Hadamard-a

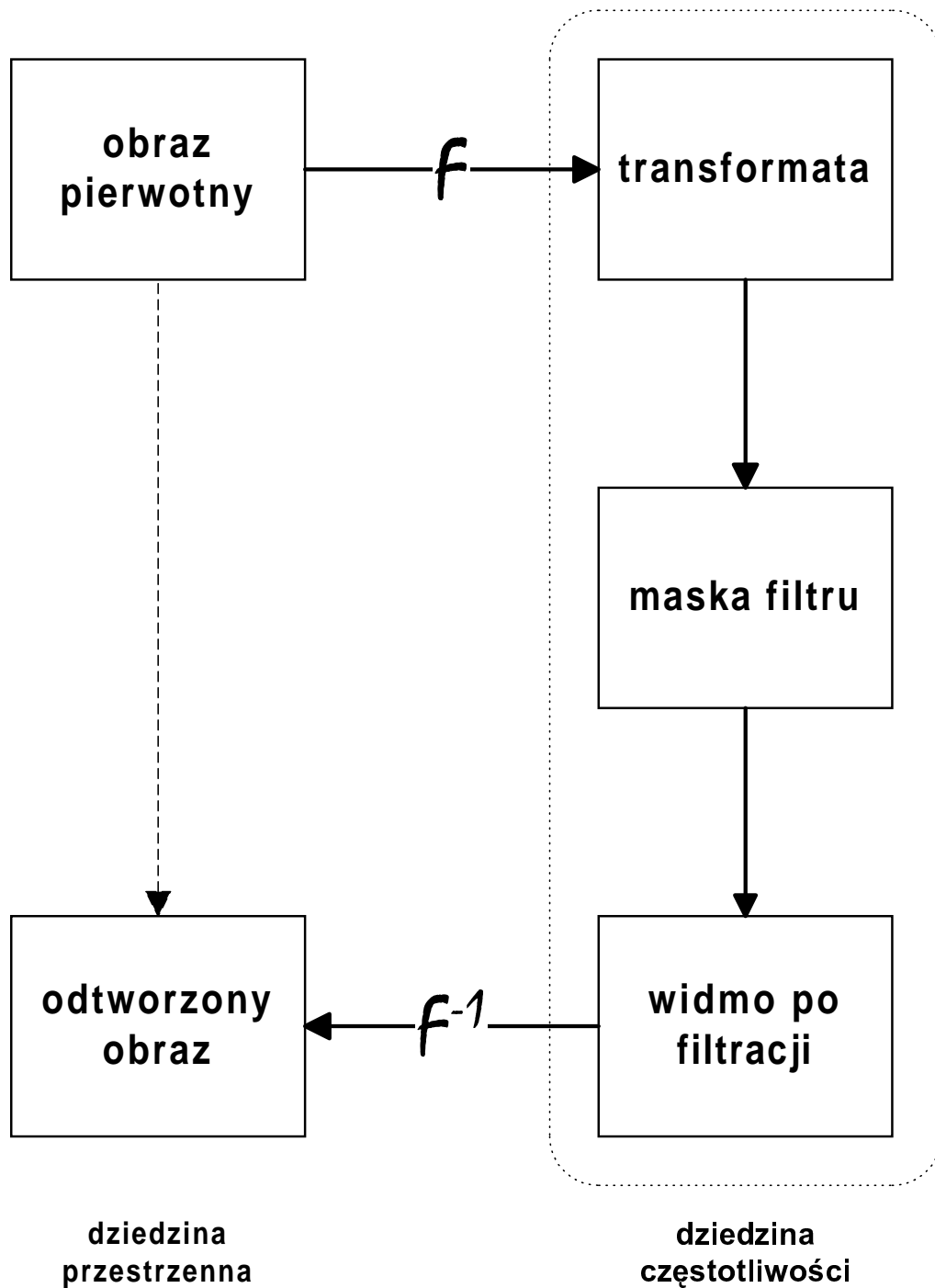
$$\mathbf{F} = \frac{1}{\sqrt{HW}} \mathbf{H}_H \mathbf{f} \mathbf{H}_W$$

$$\mathbf{f} = \frac{1}{\sqrt{HW}} \mathbf{H}_H \mathbf{F} \mathbf{H}_W$$

$$F(u, v) = \frac{1}{\sqrt{HW}} \sum_{m=0}^{H-1} \sum_{n=0}^{W-1} (-1)^{\sum_{r=0}^{b-1} m_b u_b + n_b v_b} f(m, n)$$

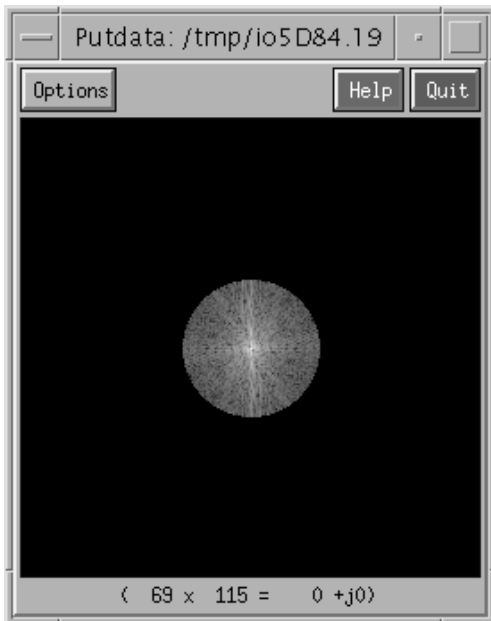
$$f(u, v) = \frac{1}{\sqrt{HW}} \sum_{m=0}^{H-1} \sum_{n=0}^{W-1} (-1)^{\sum_{r=0}^{b-1} m_b u_b + n_b v_b} F(m, n)$$

Idea filtracji w dziedzinie częstotliwości



Przykład filtracji dolno- i górnoprzepustowej

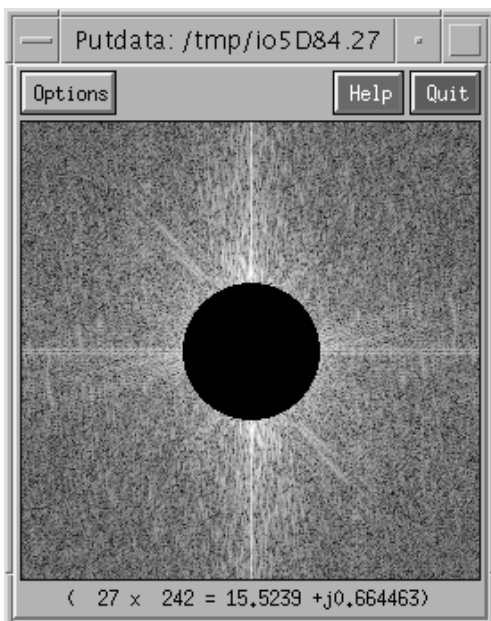
dolna część widma



wynik filtracji



górna część widma



wynik filtracji

