Lokalne transformacje obrazów

Ogólnie:

$$\Phi_{xy} : \times\{Z|(\xi,\psi)\varepsilon S_{xy}\} \rightarrow Z$$

$$g(x,y) = \Phi_{xy}((f(\xi,\psi)|(\xi,\psi)\varepsilon S_{xy}))$$

W przypadku jednorodnym:

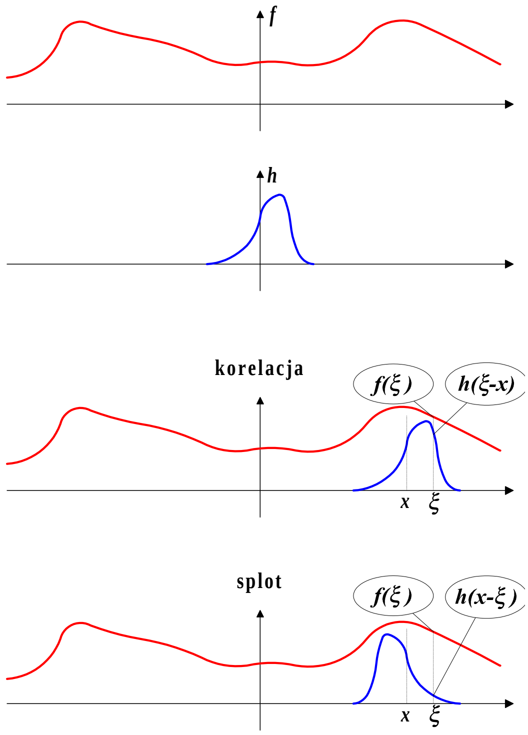
$$\Phi : \times\{Z|(\xi,\psi)\varepsilon S\} \rightarrow Z$$

$$g(x,y) = \Phi((f(\xi - x,\psi - y)|(\xi,\psi)\varepsilon S))$$

Transformacja liniowa:

$$\Phi(af + bg) = a\Phi(f) + b\Phi(g)$$

Splot a korelacja



Korelacja:

$$g(x,y) = \int_Y \int_X f(\xi,\psi)h(\xi - x, \psi - y)d\xi d\psi$$

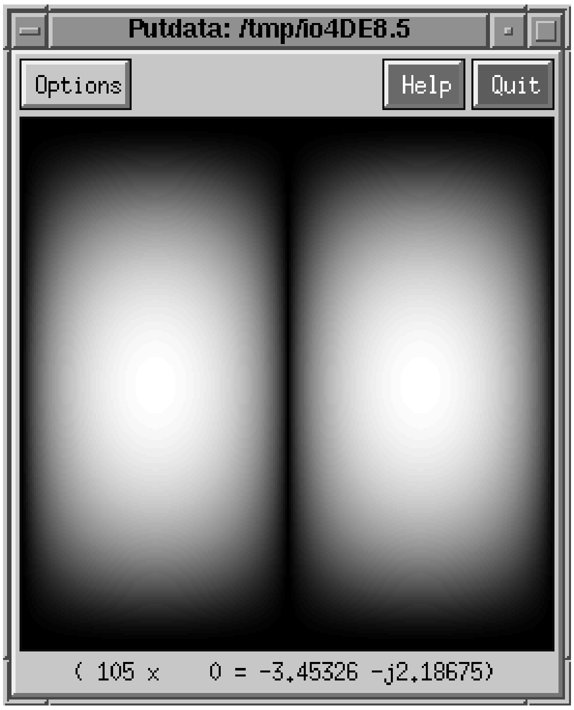
Splot:

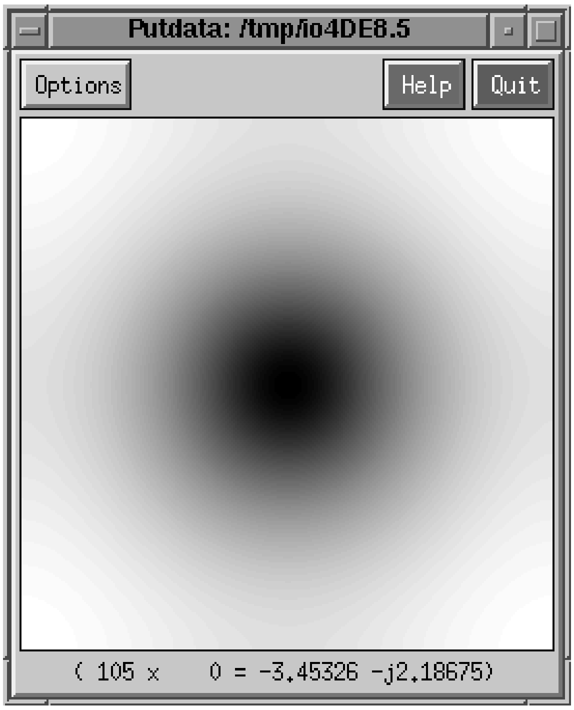
$$g(x,y) = \int_Y \int_X f(\xi,\psi)h(x - \xi, y - \psi)d\xi d\psi$$

Analiza widmowa splotu

$$\mathcal{F}(f \star h) = \mathcal{F}(f)\mathcal{F}(h)$$

$$\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 4 & 1 \\ 4 & -20 & 4 \\ 1 & 4 & 1 \end{bmatrix}$$





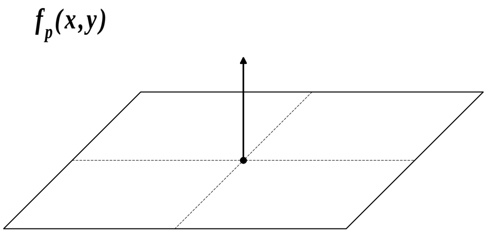
Dyskretna realizacja splotu

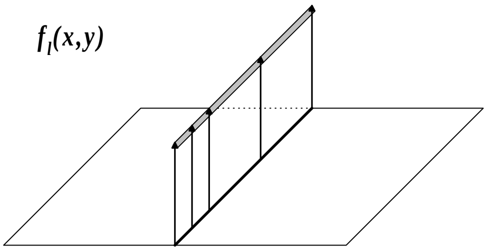
$$g(i,j) = \sum_{m=i-r}^{i+r} \sum_{n=j-r}^{j+r} f(m,n) h(i-m, j-n)$$

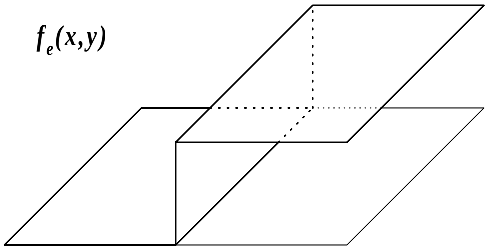
$$g(i,j) = \sum_{p=-r}^{r} \sum_{q=-r}^{r} f(i+p, j+q) h(-p,-q)$$

```
/*********************************************/
  for(i=0; i<H; i++)     /* wiersze obrazu */
  for(j=0; j<W; j++){    /* kolumny obrazu */
    g(i,j)=0;
    for(p=-r; p<=r; p++)  /* wiersze jadra */
    for(q=-r; q<=r; q++){ /* kolumny jadra */
      g(i,j)+=f(i+p,j+q)*h(-p,-q);
    }
  }
/*********************************************/
```

Obrazy testowe dla filtrów lokalnych







$$f_p(x,y) = \delta(x,y)$$

$$f_l(x,y) = \int_{-\infty}^{+\infty} \delta(x, y-\eta)\, d\eta$$

$$f_e(x,y) = \int_{-\infty}^{x} f_l(\vartheta, y)\, d\vartheta$$

Charakterystyki liniowych filtrów lokalnych

PSF - *Point Spread Function*

$$\begin{aligned} g_p(x,y) &= (f_p \star h)(x,y) = \\ &= \int \delta(\xi,\psi) h(x-\xi, y-\psi)\, d\xi\, d\psi = \\ &= h(x,y) \end{aligned}$$

LSF - *Line Spread Function*

$$\begin{aligned} g_l(x,y) &= (f_l \star h)(x,y) = \\ &= \int h(x-\xi, y-\psi) \left(\int_{\eta=-\infty}^{+\infty} \delta(x, y-\eta)\, d\eta\right) d\xi\, d\psi = \\ &= \int_{\eta=-\infty}^{+\infty} \left(\int h(x-\xi, y-\psi)\, \delta(x, y-\eta)\, d\xi\, d\psi\right) d\eta = \\ &= \int_{\eta=-\infty}^{+\infty} h(x, y-\eta)\, d\eta \end{aligned}$$

ESF - *Edge Spread Function*

$$\begin{aligned} g_e(x,y) &= (f_e \star h)(x,y) = \\ &= \int h(x-\xi, y-\psi) \left(\int_{\vartheta=-\infty}^{x} f_l(\vartheta, y)\, d\vartheta\right) d\xi\, d\psi = \\ &= \int_{\vartheta=-\infty}^{x} \left(\int h(x-\xi, y-\psi)\, f_l(\vartheta, y)\, d\xi\, d\psi\right) d\vartheta = \\ &= \int_{\vartheta=-\infty}^{x} g_l(\vartheta, y)\, d\vartheta \end{aligned}$$

Przykłady liniowych filtrów lokalnych

Filtr jednorodny:

$$g(i,j) = \sum_{(k,l)\varepsilon S_{ij}} f(k,l)$$

Filtr dwumianowy (Gauss-a):

$$g(i,j) = \sum_{k=0}^{2r} \frac{(2r)!}{(2r-k)!k!} \sum_{l=0}^{2r} \frac{(2r)!}{(2r-l)!l!} f(i+r-k, j+r-l)$$

Filtr Laplace-a:

$$g(i,j) = f(i-1,j) + f(i+1,j) + f(i,j-1) + f(i,j+1) - 4f(i,j)$$

Własności filtru Gauss-a

1	2	1
2	4	2
1	2	1

PSF

0	0	0	0	0
0	0	0	0	0
0	0	1	0	0
0	0	0	0	0
0	0	0	0	0

0	0	0	0	0
0	1	2	1	0
0	2	4	2	0
0	1	2	1	0
0	0	0	0	0

LSF

0	0	1	0	0
0	0	1	0	0
0	0	1	0	0
0	0	1	0	0
0	0	1	0	0

0	4	8	4	0
0	4	8	4	0
0	4	8	4	0
0	4	8	4	0
0	4	8	4	0

ESF

0	0	1	1	1
0	0	1	1	1
0	0	1	1	1
0	0	1	1	1
0	0	1	1	1

0	4	12	16	16
0	4	12	16	16
0	4	12	16	16
0	4	12	16	16
0	4	12	16	16

Własności filtru Laplace-a

0	1	0
1	-4	1
0	1	0

PSF

0	0	0	0	0
0	0	0	0	0
0	0	1	0	0
0	0	0	0	0
0	0	0	0	0

0	0	0	0	0
0	0	1	0	0
0	1	-4	1	0
0	0	1	0	0
0	0	0	0	0

LSF

0	0	1	0	0
0	0	1	0	0
0	0	1	0	0
0	0	1	0	0
0	0	1	0	0

0	1	-2	1	0
0	1	-2	1	0
0	1	-2	1	0
0	1	-2	1	0
0	1	-2	1	0

ESF

0	0	1	1	1
0	0	1	1	1
0	0	1	1	1
0	0	1	1	1
0	0	1	1	1

0	1	-1	0	0
0	1	-1	0	0
0	1	-1	0	0
0	1	-1	0	0
0	1	-1	0	0

Poprawianie krawędzi

0	-1	0
-1	5	-1
0	-1	0

0	0	0	0	0
0	0	0	0	0
0	0	1	0	0
0	0	0	0	0
0	0	0	0	0

PSF

0	0	0	0	0
0	0	-1	0	0
0	-1	5	-1	0
0	0	-1	0	0
0	0	0	0	0

0	0	1	0	0
0	0	1	0	0
0	0	1	0	0
0	0	1	0	0
0	0	1	0	0

LSF

0	-1	3	-1	0
0	-1	3	-1	0
0	-1	3	-1	0
0	-1	3	-1	0
0	-1	3	-1	0

0	0	1	1	1
0	0	1	1	1
0	0	1	1	1
0	0	1	1	1
0	0	1	1	1

ESF

0	-1	2	1	1
0	-1	2	1	1
0	-1	2	1	1
0	-1	2	1	1
0	-1	2	1	1

Przykłady nieliniowych filtrów lokalnych

Lokalne minimum:

$$g(i,j) = min\{f(k,l) | (k,l) \varepsilon S_{ij}\}$$

Lokalne maksimum:

$$g(i,j) = max\{f(k,l) | (k,l) \varepsilon S_{ij}\}$$

Filtr medianowy:

$$g(i,j) = med(f(k,l) | S_{ij})$$

Przypomnienie:

$$z = med(f(\zeta)) \Leftrightarrow \int_{\zeta=-\infty}^{z} f(\zeta) d\zeta = \frac{1}{2} \int_{\zeta=-\infty}^{\infty} f(\zeta) d\zeta$$

Własności lokalnego minimum

MIN_1

PSF

0	0	0	0	0
0	0	0	0	0
0	0	1	0	0
0	0	0	0	0
0	0	0	0	0

0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0

LSF

0	0	1	0	0
0	0	1	0	0
0	0	1	0	0
0	0	1	0	0
0	0	1	0	0

0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0

ESF

0	0	1	1	1
0	0	1	1	1
0	0	1	1	1
0	0	1	1	1
0	0	1	1	1

0	0	0	1	1
0	0	0	1	1
0	0	0	1	1
0	0	0	1	1
0	0	0	1	1

Własności filtru medianowego

MED_1

PSF

0	0	0	0	0
0	0	0	0	0
0	0	1	0	0
0	0	0	0	0
0	0	0	0	0

0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0

LSF

0	0	1	0	0
0	0	1	0	0
0	0	1	0	0
0	0	1	0	0
0	0	1	0	0

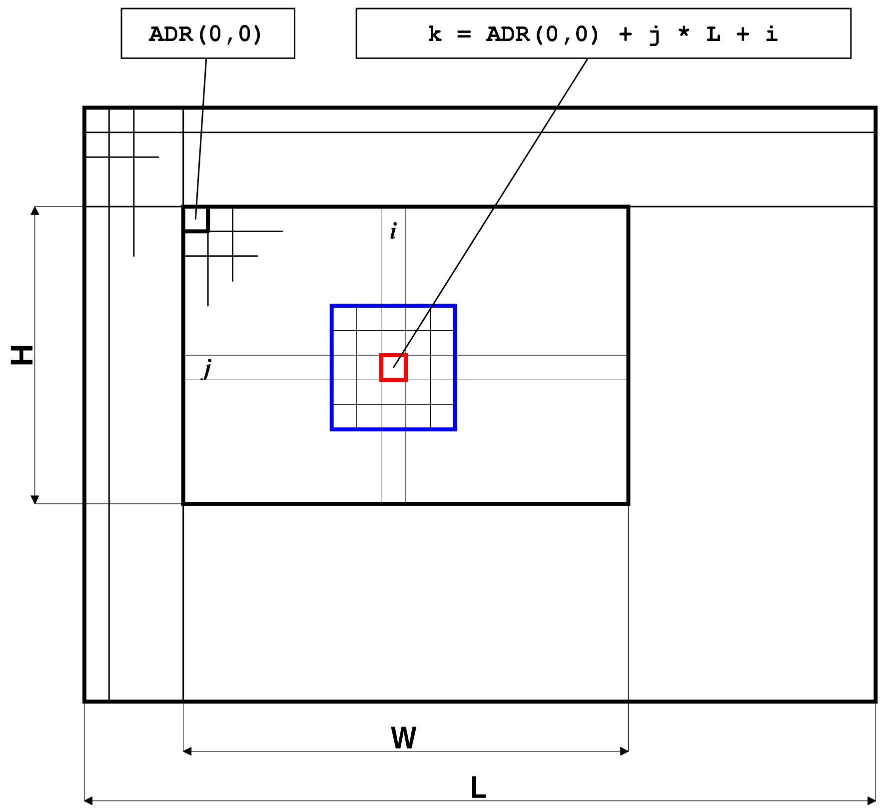
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0

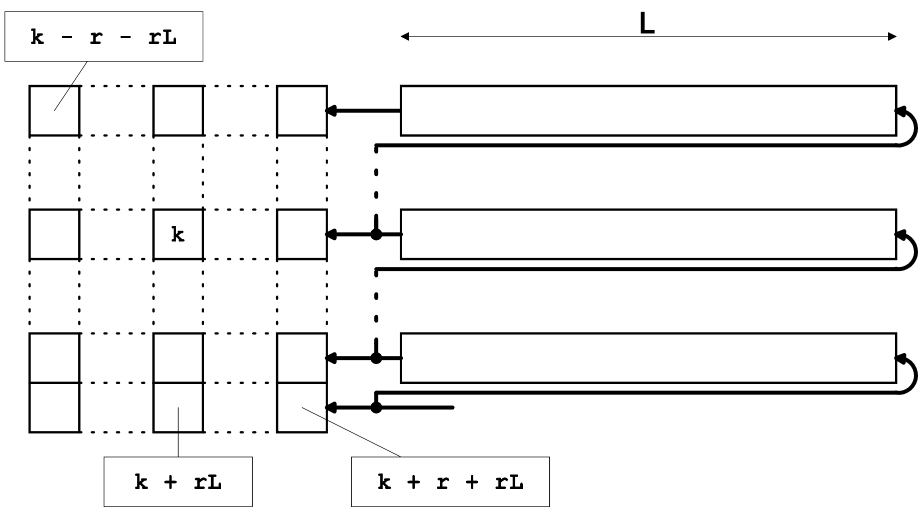
ESF

0	0	1	1	1
0	0	1	1	1
0	0	1	1	1
0	0	1	1	1
0	0	1	1	1

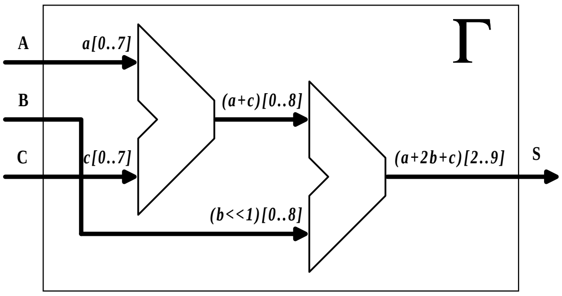
0	0	1	1	1
0	0	1	1	1
0	0	1	1	1
0	0	1	1	1
0	0	1	1	1

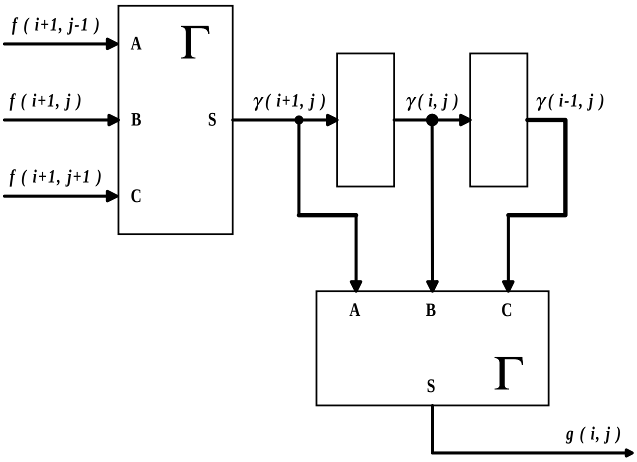
Potokowa realizacja filtracji lokalnych





Sprzętowa realizacja filtru Gauss-a



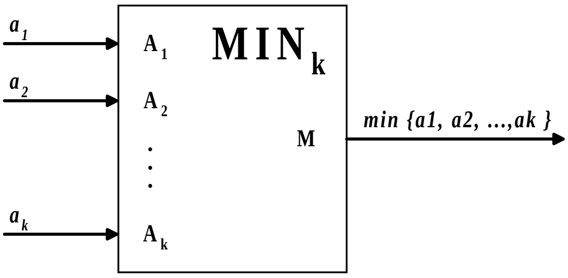


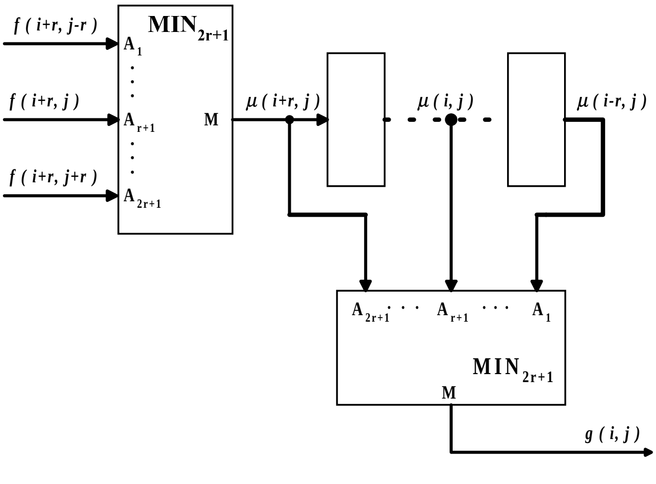
$$\Gamma(a,b,c) = a + 2b + c$$

$$\gamma(i,j) = \Gamma(f(i,j-1), f(i,j), f(i,j+1))$$

$$g(i,j) = \Gamma(\gamma(i-1,j), \gamma(i,j), \gamma(i+1,j))$$

Sprzętowa realizacja minimum lokalnego

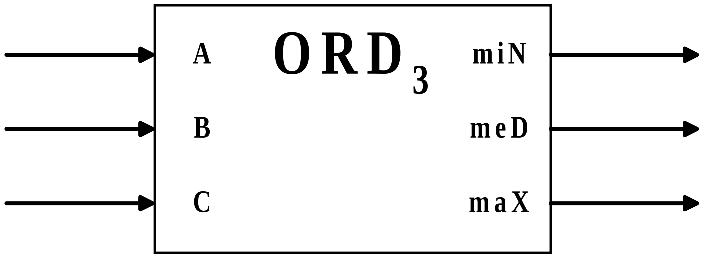


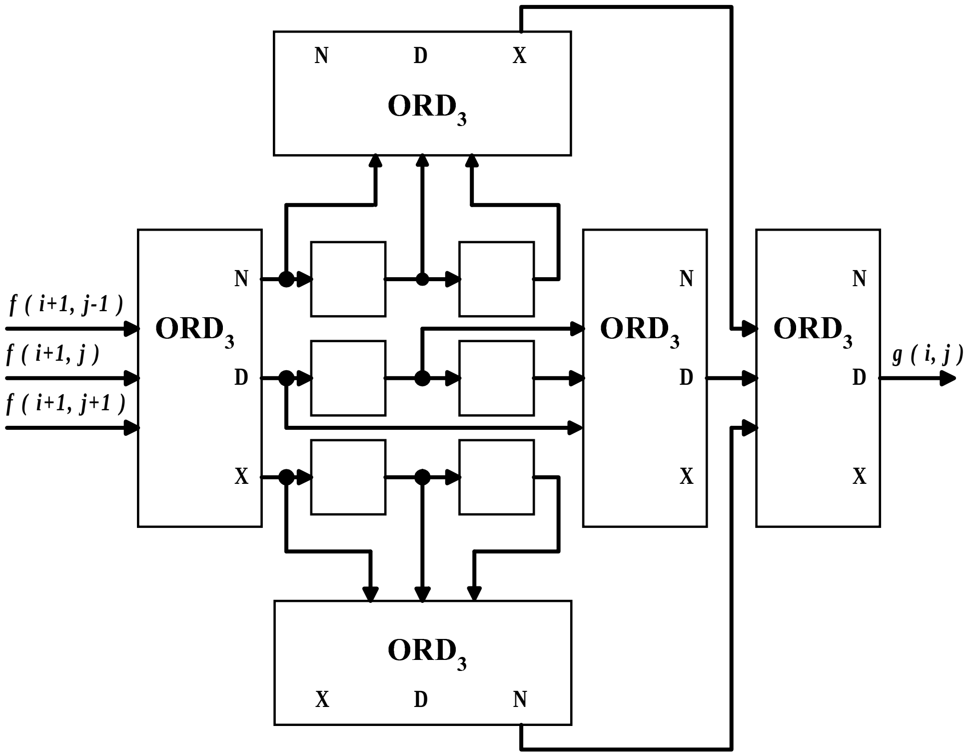


$$\mu(i,j) = MIN_{2r+1}\{f(i,j-r),\ldots,f(i,j),\ldots,f(i,j+r)\}$$

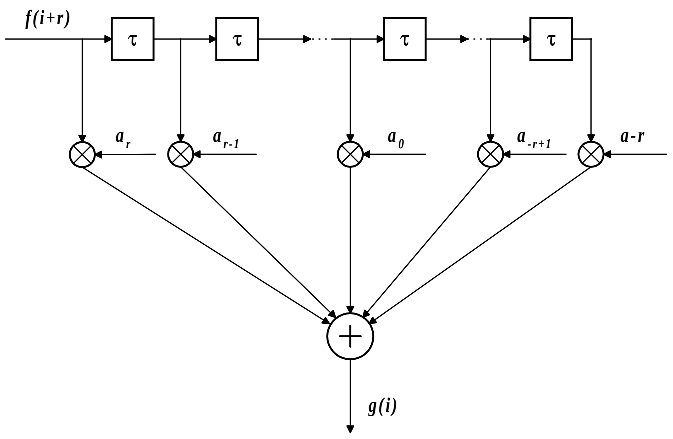
$$g(i,j) = MIN_{2r+1}\{\mu(i,j-r),\ldots,\mu(i,j),\ldots,\mu(i,j+r)\}$$

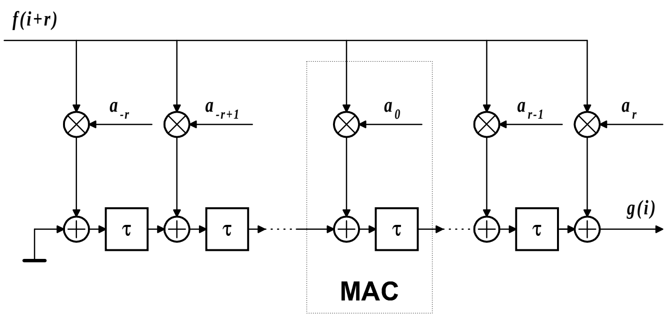
Sprzętowa realizacja filtru medianowego





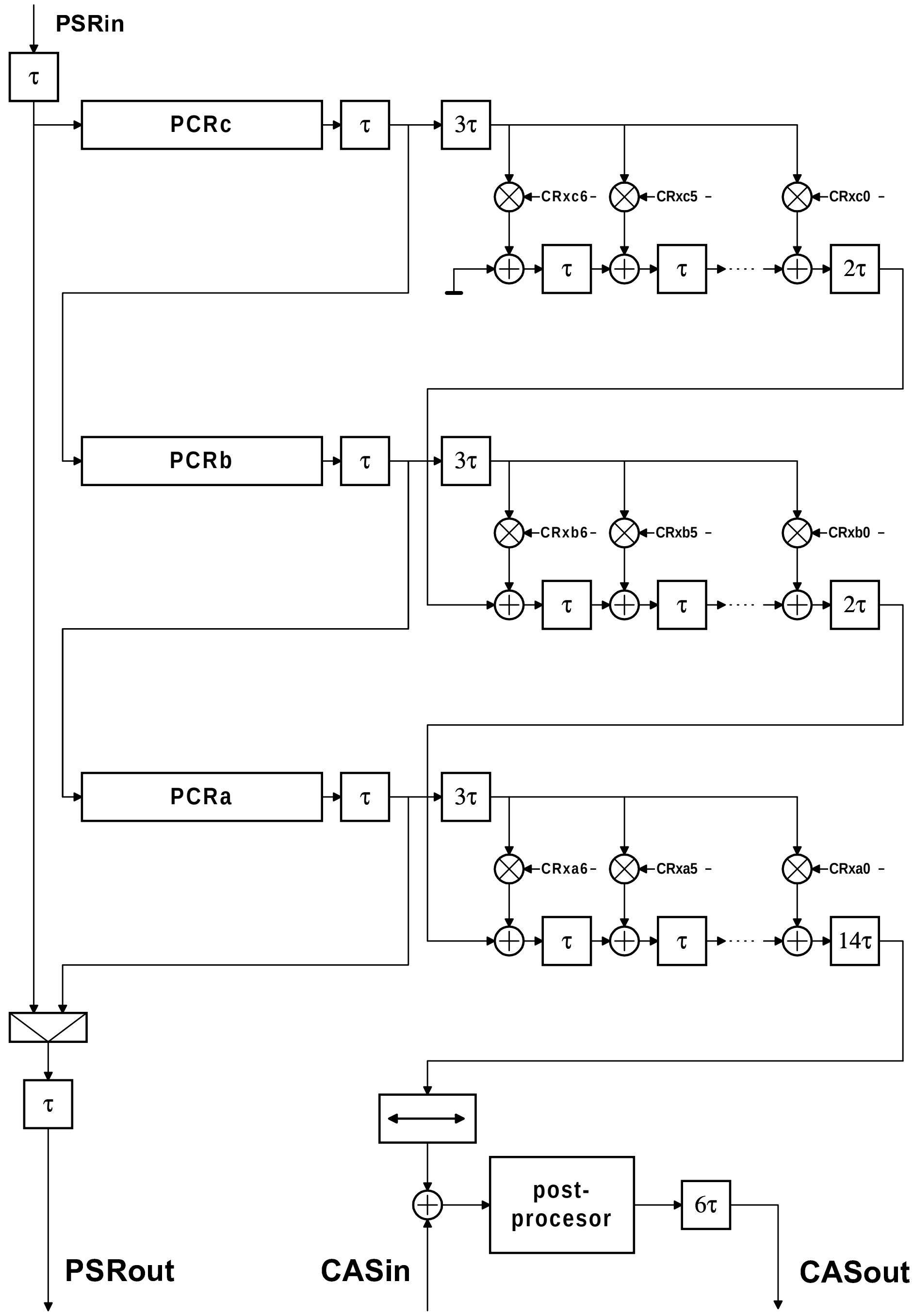
Realizacja filtru FIR (*Finite Response Filter*)





$$g(i) = \sum_{k=-r}^{r} a_k f(i+k)$$

Koprocesor splotowy IMS A110



Przykłady implementacji filtrów liniowych przy pomocy IMS A110

Filtr jednorodny:

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\frac{p}{2^q} \approx \frac{1}{9} \Rightarrow p = 57 \; ; \; q = 9$$

$$\frac{1}{2^9}\begin{bmatrix} 57 & 57 & 57 \\ 57 & 57 & 57 \\ 57 & 57 & 57 \end{bmatrix}$$

Filtr Laplace-a:

$$\frac{1}{6}\begin{bmatrix} 1 & 4 & 1 \\ 4 & -20 & 4 \\ 1 & 4 & 1 \end{bmatrix} \approx \frac{1}{2^5}\begin{bmatrix} 5 & 22 & 5 \\ 22 & -108 & 22 \\ 5 & 22 & 5 \end{bmatrix}$$

Filtracja odwrotna

Obraz zniekształcony liniowo:

$$f(x,y) = (g \star h)(x,y)$$

$$F(u,v) = G(u,v)H(u,v)$$

Filtr odwrotny w dziedzinie częstotliwości:

$$G(u,v) = \frac{1}{H(u,v)}F(u,v)$$

Filtr o ograniczonym wzmocnieniu:

$$G = \frac{H^\star}{HH^\star + B^2}F$$

Rozplot (*deconvolution*)

Algorytm rozplotu

$$g_{k+1} = g_k + (f - (g_k \star h)) \; ; \; g_0 = f$$

Przyklady:

Niedokładne zogniskowanie obiektywu:

$$h(x,y) = \begin{cases} \frac{1}{\pi R^2} & \Leftrightarrow \quad x^2 + y^2 \leq R^2 \\ 0 & \Leftrightarrow \quad x^2 + y^2 > R^2 \end{cases}$$

Ruch obiektu (lub kamery) wzdłuż osi x:

$$h(x,y) = \begin{cases} \frac{1}{2r}\delta(y) & \Leftrightarrow \quad -r \leq x \leq +r \\ 0 & \Leftrightarrow \quad x < -r \vee x > +r \end{cases}$$